

Differentiation

$$1. f(x) = \frac{3x-1}{x+2}, \quad x \neq -2$$

from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{3x-1}{x+2} \quad f(x+h) = \frac{3(x+h)-1}{x+h+2}$$

$$= \frac{3x+3h-1}{x+h+2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{3x+3h-1}{x+h+2} - \frac{3x-1}{x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+2)(3x+3h-1) - (x+h+2)(3x-1)}{h(x+h+2)(x+2)}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 3xh - x + 6x + 6h - 2 - (3x^2 + 3xh + 6x - x + 2x)}{h(x+h+2)(x+2)}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 3xh - x + 6x + 6h - 2 - 3x^2 - 3xh - 6x + x + 2x}{h(x+h+2)(x+2)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{6h+h}{x^2 + 4x + 4 + xh + 2h} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{7h}{x^2 + 4x + 4 + xh + 2h} \right)$$

$$= \lim_{h \rightarrow 0} \frac{7}{x^2 + 4x + 4 + xh + 2h}$$

$$f'(x) = \frac{7}{x^2 + 4x + 4}$$